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FINAL REPORT
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NONLINEAR WAVES AND INVERSE SCATTERING
M.J. ABLOWITZ, PRINCIPAL INVESTIGATOR

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The research funding for this grant has been used to support Professors Mark J. Ablowitz and A.S. Fokas and their associates. The P.I. has been working in this field for over twenty years, with the central theme being the understanding of the nonlinear phenomena arising in physical problems. The solution to a number of these problems involves the Inverse Scattering Transform (I.S.T.). This is a method which employs inverse scattering theory in order to linearize the given nonlinear equation. I.S.T. has led to new developments in both fields: inverse scattering and nonlinear wave equations. Listed below are some of the problems studied and a short description of results.

Multidimensional Nonlinear Evolution Equations and Inverse Scattering: There are a number of physically interesting nonlinear wave equations for which the inverse scattering transform provides a linearization. Well known systems include the Kadomtsev-Petviashvili, Davey-Stewartson and Self-Dual Yang-Mills equations. The d-bar method of inverse scattering is an effective tool to analyze the solutions to these problems. We have uncovered an important feature of many of the multidimensional nonlinear equations solvable by Inverse Scattering. Namely the fact that they are nonlocal, allows a variety of possible boundary conditions that can be imposed. Different boundary value problems require different inverse scattering transforms for the solution. The role of boundary conditions are important in understanding why exponentially decaying multidimensional soliton solutions exist for some of these systems (i.e. one version of the Davey-Stewartson System). The d-bar method can be employed to analyze Inverse problems in any number of dimensions.

Semi-Infinite and Forced Nonlinear Equations.

Recent research has shown that moving pressure distributions at critical velocities can induce a soliton radiation field under the assumption of Long Waves. There is direct application of these ideas to ships moving in shallow canals at suitable velocities. There has been considerable experimental and numerical simulations of the phenomena. Our analysis has shown that there is a close relationship between nonlinear equations with generalized (e.g. Dirac delta and derivatives) function forcing and their counterparts on the semi-infinite line. This opens up a real possibility that certain forced nonlinear equations will be amongst those solved by the IST method.

Numerically Induced Chaos

We have been studying a class of non-linear equations and their discrete approximations. Our numerical simulations have shown that associated with standard numerical schemes of well known nonlinear wave equations (the prototype is the nonlinear Schrodinger equation) are *solutions* which exhibit a weak form of temporal chaos. This chaos disappears if the mesh is refined to a sufficiently fine degree, but far more than one would expect. Schemes which have been developed by the IST method do not exhibit such spurious chaos. IST schemes are only second order accurate, but yet perform far better than do higher order accurate methods such as spectral schemes (which are infinite order accurate!). Our analytical studies show that proximity to homoclinic orbits in the underlying nonlinear equation is the mechanism responsible for the observed chaos. This surprising situation occurs for many of the well known nonlinear wave equations in one dimension and is likely to occur in higher dimensional problems as well.

Cellular Automata and Solitons

We have been studying a class of cellular automata (CA) which admit a wide variety of coherent particle-like solutions. Computer scientists have shown that some of these CA arise in the study of computation. In our analysis we have (a) shown that these particles will interact solitonically under appropriate circumstances.
(b) Characterized a wide class of periodic particles and have shown that they satisfy a linear difference equation. (c) Generalized the CA

to higher dimensions and have shown that particle like solutions can be constructed and even interact solitonically under suitable circumstances.

It is important to note that these systems are not reversible and as such this is a significant difference between soliton CA and the well known soliton theory in continuous and discrete media.

Following is a list of published papers in 1987-1989.

A. Papers Published:

(1) An Example of a $\bar{\partial}$ Problem Arising in a Finite Difference Contex: Direct and Inverse Problem for the Discrete Analogue of the Equation, O. Ragnisco, P.M. Santini, S. Chitlaru-Briggs and M.J. Ablowitz, J. Math. Phys., 28 1987), 777.

(2) Note on Solutions to a Class of Nonlinear Singular Integro-differential equations, M.J. Ablowitz, A.S. Fokas and M.D. Kruskal, Phys. Lett. A, Vol. 120, (5) (1987), 215-218.

(3) Solutions of Multidimensional Extensions of the Anti-Self-Dual Yang-Mills Equations, M.J. Ablowitz, D.G. Costa and K. Tenenblat, Stud. in Appl. Math., 77 (1987).

(4) Topics Associated with Nonlinear Evolution Equations and Inverse Scattering in Multidimensions, M.J. Ablowitz, edited by M. Lakshmanan, Proceedings of "Solitons", Winter School, Tiruchirapalli, India (January 1987), INS #76, preprint.

(5) Numerical Simulation of the Modified Korteweg-deVries Equation, Thiab R. Taha and M.J. Ablowitz, 6th International Symposium on Computer Methods in PDE's Proceedings, 1987.

(6) On the Initial Value Problem for a Class of Nonlinear Integral Evolution Equations Including the Sine-Hilbert Equation, P.M. Santini, M.J. Ablowitz and A.S. Fokas, J. Math. Phys., 28 (10) (October 1987).

(7) Davey-Stewartson I System: A Quantum (2+1) Dimensional Integrable System, C.L. Schultz, M.J. Ablowitz and D. Bar Yaacov, Phys. Rev. Lett., Vol. 59, No. 25 (December 1987).

(8) A Rule for Fast Computation and Analysis of Soliton Automata, T.S. Papatheodorou, M.J. Ablowitz and Y.G. Saridakis, Stud. in Appl. Math. 79, 1988.

(9) Analytical and Numerical Aspects of Certain Nonlinear Evolution Equations IV, Numerical, Modified Korteweg-de Vries Equation, T.R. Taha and M.J. Ablowitz, J. Comp. Physics, Vol. 77, No. 2 August 1988.

(10) A Method of Linearization for Painleve Equations: Painleve IV, V, A.S. Fokas, U. Mugan and M.J. Ablowitz, Physica D 30, PP. 247-283, 1988.

(11) Action-Angle Variables and Trace Formula For D-Bar Limit Case of Davey-Stewartson I, C.L. Schultz and M.J. Ablowitz, Phys. Lett. A., Vol. 135, No. 8,9, March 1989.

(12) Numerically Induced Chaos in the Nonlinear Schrodinger Equation, B. Herbst and M.J. Ablowitz, Phys. Rev. Lett., Vol. 61, No. 18, 1989.

(13) Strong Coupling Limit of Certain Multidimensional Nonlinear Wave Equations, M.J. Ablowitz and C.L. Schultz, Stud. in Appl. Math. 80, 1-10, June 1989.

(14) Forced Nonlinear Evolution Equations and the Inverse Scattering Transform, A.S. Fokas and M.J. Ablowitz, Stud. in Appl. Math. 80, 253-272, 1989.

(15) Hodograph Transformations of Linearizable Partial Differential Equations, P.A. Clarkson, A.S. Fokas and M.J. Ablowitz, SIAM Jrnl. on Appl. Math. 49, No. 4, 1188-1209, 1989.

(16) Interaction of Simple Particles in Soliton Cellular Automata, A.S. Fokas, E.P. Papadopoulou, Y.G. Saridakis and M.J. Ablowitz, Studies in Applied Math. 81, 153-180, 1989.

B. Papers Submitted:

(1) Nonlinear Evolution Equations and Cellular Automata, M.J. Ablowitz, INS#111, to be published Proceedings for Singular Behavior and Nonlinear Dynamics, held in Samos, Greece, August 1988.

(2) Painleve Equations and the Inverse Scattering and Inverse Monodromy Transforms, M.J. Ablowitz, INS#105, September 1988, to be published Proceedings on Solitons in Physics and Mathematics, Institute of Math and Its Applications.

(3) Nonlinear Evolution Equations, Inverse Scattering and Cellular Automata, M.J. Ablowitz, INS#114 to be published Proceedings of IMA Workshop on Solitons in Nonlinear Optics and Plasma Physics, Minneapolis, Minnesota, November 1988.

(4) On Numerical Chaos in the Nonlinear Schrodinger Equation, B. Herbst and M.J. Ablowitz, to appear Proceedings of Workshop on Complete Integrability, Orleon, France, INS#120, January 1989.

(5) Solitons, Inverse Problems and Nonlinear Equations, INS#118, M.J. Ablowitz, February 1989, to be published Journal of Computational and Applied Mathematics.

(6) Nonlinear Evolution Equations, Solitons, Chaos and Cellular Automata, M.J. Ablowitz, B.M. Herbst and J.M. Keiser, INS#123, April 1989. To appear Proceedings of International conference on Nonlinear Problems, Shangai, China.

(7) Nonlinear Wave Propagation, M.J. Ablowitz, INS#113, January 1989. To appear Encyclopedia of Physics (accepted).

(8) On Homoclinic Structure and Numerically Induced Chaos for The Nonlinear Schrodinger Equations, M.J. Ablowitz and B.M. Herbst, SIAM Journal in Applied Mathematics (accepted), PAM #1.
Note: INS denotes Institute for Nonlinear Studies Report (Clarkson University). PAM denotes Program in Applied Mathematics report (University of Colorado).

C. Books (submitted): NONE

D. Books (published): NONE

E. Patents filed: NONE

F. Patents granted: NONE

G. Invited Presentations:

"Solitons", Winter School, Tiruchirapalli, India, January 1987.

Institute for Mathematics and its Applications, University of Minnesota, IMA Program in Inverse Problems, Minneapolis, Minnesota, January 1987.

Virginia Polytechnic Institute, Department of Mathematics, Blacksburg, Virginia, February 1987.

4th Workshop Nonlinear Evolution Equations and Dynamical Systems, June 11-25, 1987, Montpellier, France.

AMS Thirty-Fifth Summer Research Institute, Bowdoin College, Brunswick, Maine, July 6-24, 1987.

Workshop on Nonlinear Waves held at the Institute for Applied Mathematics, University of Minnesota, Minneapolis, Minnesota, July 24-27, 1987.

National Science Foundation, Washington, D.C., October 9, 1987.

SIAM 35th Anniversary Meeting, Denver, Colorado, October 12-15, 1987.

M.J. Ablowitz, "Recent Results on Solitons and Nonlinear Evolution Equations", Integrable Systems and Applications Workshop, Oleron, France, June 1988.

M.J. Ablowitz, "Recent Results Regarding Solitons and the Inverse Scattering Transform", International Workshop on Nonlinear Evolution Equations: Integrability and Spectral Methods, Como, Italy, July 4-15, 1988.

M.J. Ablowitz, "New Results on Solitons and the Inverse Scattering Transform", International Conference "Singular Behavior and Nonlinear Dynamics", Samos, Greece, August 18-26, 1988.

M.J. Ablowitz, "Painleve Equations and the Inverse Scattering and Inverse Monodromy Transform", IMA Workshop on Mathematical Theory of Solitons, Minneapolis, MInnesota, September 12-16, 1988.

M.J. Ablowitz, "Solitons in Continuous and Discrete Media", Math Methods in Plasma Physics, Cornell Univ., Oct. 20-23, 1988.

M.J. Ablowitz, "Inverse Scattering and Cellular Automata", Applications of Solitons in Physics, IMA, Univ. of Minn., Nov. 1988.

M.J. Ablowitz, "Solitons, Nonlinear Evolution Equations and Cellular Automata", Rutgers University, Department of Mathematics Colloquium, March 3, 1989.

M.J. Ablowitz, "Solitons, Nonlinear Evolution Equations, Inverse Scattering and Cellular Automata", Shanghai, China, International Conference on Nonlinear Physics, April 24-30, 1989.

M.J. Ablowitz, "Aspects of Solitons and Computation", Computing Center of Academia Sinica, Beijing, China, May 6, 1989.

M.J. Ablowitz, "Solitons and Computation", Workshop on Hamiltonian Systems and Inverse Spectral Transform, University of Montreal, Montreal, Canada, Oct. 20-26, 1989.

M.J. Ablowitz, "Solitons and All That Nonlinear Stuff", Mathematics Department, Colorado School of Mines, Nov., 1989.

H. Graduate Students and Postdoctorals Supported: James Keiser, Elizabeth Ryan (graduate students).